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THE REVIEW OF Z TRANSFORM AND ITS APPLICATION IN ENGINEERING

R. A. Naphade*, R. M. Jetwani & U. Y.Gawarguru
Department of Applied Science & Humanities, MGI-COET, Shegaon

ABSTRACT

It deals with a review of Z Transform and specific region of convergence represents for it. Most important Z Transform properties have been mentioned. This paper also insights the Inverse Z Transform and provides multiple methods to find the same. The use of Z Transform is to convert a simple or a complicated sequence in to a corresponding frequency domain equivalent.

***Keywords**: Z Transform, Inverse Z Transform, Convolution, Region of Convergence*

I. INTRODUCTION

This paper deals with the introduction to Z Transform is and its application. A method for solving linear constant coefficient difference equation by Laplace Transform was introduced to graduate engineering students by Gardner and Barnes in the early 1940's. They applied their procedure which was based on jump function to transmission lines and application involving Bessel function. This approach is quite complicated and in separate attempt simplify matters, a transform of sampled signal or sequence was defined in 1947 by W. Hyrewicz was later denoted in 1952 as "Z transform" by Ragazzini and Zadeh in sampled data control group at Colombia university. In Mathematical studies have a number of transform such as the Laplace Transform, the Fourier Transform, etc. But, both Laplace and Fourier Transforms are continuous functions and cannot be used to study discrete systems. There is a way to convert the continuous Fourier Transform into its discrete equivalent by finding the Discrete Fourier Transform (DFT) but Fourier Transform cannot be used to study discrete systems because it is continuous. Linear systems in which the input signals are in the form of discrete pulses are called as 'Linear Time Variants'. A discrete system is expressible as a difference equation and its solution are found using Z transform. For the analysis of such systems, we need Z Transform. The Z Transform will convert a sequence to corresponding frequency domain equivalent.The basic idea of the Z-transform was known to Laplace. The advanced Z-transform was later developed and popularized by E. I. Jury.

II. SEQUENCE

Z transform is defined for a specific sequence. If 'n' items are arranged according to a specific rule, then that arrangement is known as a sequence. Thus, in general, an ordered set of real or complex numbers is called a sequence. In this paper, a general sequence will be represented by {x (n)} with n being its index and x (n) being the term.

III. Z TRANSFORM

Definition:

Let x (n) be any sequence, then Z transform is defined as,

$$Z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \text{……….} (1)$$

Where z is to be taken complex variable.
This expression is sometimes referred to as two sided z transform since the summation is over all integers.
If x (n) =0, n<0 then the Z transform is

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$$Z\{x(n)\} = X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \text{ ………. (2)}$$

Z transform exists only for the values of z for which series if (1) and (2) converges.

The series of equation (2) is said to be converge absolutely when the series of real numbers $\sum_{n=0}^{\infty} \left| x(n) z^{-n} \right|$ Converges. It is known that a series converges absolutely also converges.

IV. SOME STANDERD Z TRANSFOMRS

1. DiscreteUnit Impulse defined as
   $$\delta(n) = x(n) = 1 \quad , n = 0$$
   $$= 0 \quad , otherwise$$
   Here, $X(z) = 1$
2. DiscreteUnit step defined as
   $$u(n) = x(n) = 1 \quad , n \geq 0$$
   $$= 0 \quad , otherwise$$
   Here, $X(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}} = \frac{z}{z-1}$
3. If $x(n) = a^n$ then, $X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \frac{z}{z-a}$
4. If $x(n) = n^p (n \geq 0, p > 0)$ then $X(z) = \sum_{n=0}^{\infty} n^p z^{-n} = -z \frac{d}{dz}\{Z(n^{p-1})\}$

V. REGION OF CONVERGENCE

Every Z Transform is defined over a region. This region is known as the region of convergence. Consider the following cases of region of convergence is

- $|z| < a$

Since $z = x + iy$ is any complex number& $|z| = \sqrt{x^2 + y^2}$ z.

Now if $|z| < a$, then $\sqrt{x^2 + y^2} < a$

$\therefore x^2 + y^2 < a^2$

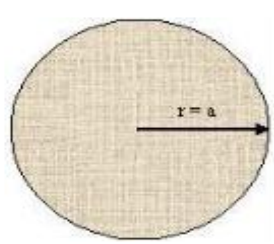

Figure 1: $|z| < a$

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The equation $x^2 + y^2 = a^2$ is the equation of a standard circle. Thus, $x^2 + y^2 < a^2$ will represent the region inside the circle of radius a.

- $|z| > a$

Since $z = x + iy$ is any complex number & $|z| = \sqrt{x^2 + y^2}$.

Now, if $|z| > a$, then $\sqrt{x^2 + y^2} > a \therefore x^2 + y^2 > a^2$

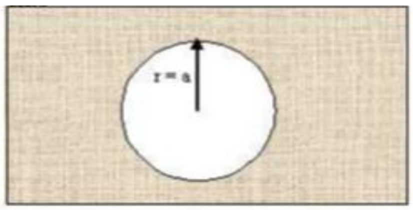

Figure 2: $|z| > a$

The equation $x^2 + y^2 = a^2$ is the equation of a standard circle. Thus, $x^2 + y^2 > a^2$ will represent theregionoutside the circle of radius a

VI. PROPERTIES OF Z TRANSFORM

Linearity property:

If α and β are constants and $x_1(n)$ & $x_2(n)$ be two discrete functions then, $Z\{\alpha x_1(n) + \beta x_2(n)\} = \alpha Z\{x_1(n)\} + \beta Z\{x_2(n)\}$

Damping rule:

If $Z\{x(n)\} = X(z)$ Then $Z\{a^{-n}x(n)\} = X(az)$ and $Z\{a^n x(n)\} = X\left(\frac{z}{a}\right)$.

Shift property:

If $Z\{x(n)\} = X(z)$ then $Z\{x(n-m)\} = z^{-m}X(z)$

Multiplication by k:

If, $Z\{x(n)\} = X(z)$ then $Z\{nx(n)\} = -\frac{d}{dz}X(z)$

Convolution property:

If x (n) and y (n) are two sequences, then

$Z\{x(n) * y(n)\} = X(z) * Y(z)$

VII. INITIAL & FINAL VALUE THEOREM

- **INITIAL VALUE THEOREM**

If $Z\{x(n)\} = X(z)$ Then $x(0) = \lim_{z \to \infty} X(z)$

- **FINAL VALUE THEOREM:**

If $Z\{x(n)\} = X(z)$ Then $\lim_{n\to\infty} x(n) = \lim_{z\to 1}(z-1).X(z)$

VIII. INVERSE Z TRANSFORM

The process of obtaining the sequence x (n) corresponding to a z transform X (z) is known as inversion

$Z^{-1}\{X(z)\} = x(n)$

IX. METHODS TO FIND Z TRANDFORM:

The following methods can be used to find the Inverse Z Transform of a function in z.

Long division:

Since By definition Of Z Transform: $X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} = x(0) + x(1)z^{-1} + x(2)z^{-2} + ...$ to find its inverse z transform expandX (z) in proper power series and collect the coefficient of z^{-n} to get x (n).This power series whose coefficients are the terms of the sequence.

Bionomial expansion:

This method of finding the Inverse Z Transform is relatively advanced. To apply this method, a factor is to be taken in common from the denominator depending upon the region of convergence so that the denominator is of theform 1 –x where |x|<1 and then, binomial theorem will be used.

This method can be better explained by an example.

Let $F(z) = \frac{1}{z-a}$ with region of convergence |z| < |a|. As explained earlier, the denominator will be expressed as 1 – x where |x|<1. Now as the denominator is $z-a$ with the region of convergence as |z| < |a|,

$$F(z) = \frac{1}{a\left(\frac{z}{a} - 1\right)}$$

$F(z) = -\frac{1}{a}\left(1 - \frac{z}{a}\right)^{-1}$ Thus, $F(z)$ has now been modified to a situation wherein it is possible to apply the method of binomial expansion and by using the definition of Z Transform, it will then be possible to get back the original sequence.

Using the method of binomial expansion

$$F(z) = -\frac{1}{a} - \frac{z}{a^2} - \frac{z}{a^2} - \frac{z}{a^3}\cdots\cdots$$

$$F(z) = \sum_{k=-\infty}^{0} -a^{k-1}z^{-k}$$

$$Z^{-1}\{F(z)\} = -a^{k-1} \quad ,k \le 0$$

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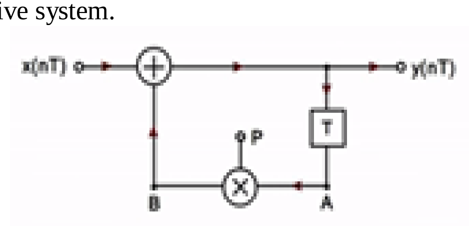
Partial fractrion:

Given $X(z)$ find partial fraction of If $\frac{X(z)}{z}$ and multiply the resulting expansion by z and find the inversion. If the degree of the numerator is greater than or equal to the degree of the denominator, the function X (z) is first divided by z and this is continued until the time the degree of the numerator polynomial is lesser than the degree of the denominator polynomial.

X. APPLICATION OF Z TRANSFORM

A crucial advantage of Z transform is that it helps convert complex sequences into their corresponding frequency domain equivalents. The Z transform is a very effective tool in the analysis discrete time signals. It is also used to determine the frequency response of these discrete time signals. Z transform plays a most important role in digital signal processing.
Consider an example
Find the impulse response of recursive system.



Solution: Assuming initially relaxed system.
Let difference equation is

$$y(nT) = x(nT) + py(nT - T)$$

If $x(nT) = \delta(nT)$

$$y(nT) = \delta(nT) + py(nT - T)$$

For an initially relaxed System $y(nT) = 0 \quad , n < 0$,and

$$y(0) = \delta(0) + py(-T) = 1 + 0 = 1$$
$$y(T) = \delta(T) + py(0) = 0 + p * 1 = p$$
$$y(2T) = \delta(2T) + py(T) = 0 + p * p = p^2$$
.
.
$$y(nT) = u(nT) p^n$$

XI. CONCLUSIONS

This paper consisted of an overview of what Z Transform and their regions of convergence represent. As due to its different properties ofZ Transformto analyze discrete signals is very important whereas Fourier Transform and Laplace Transform, both being continuous. For very branchunder signal processing doesmakeeffective use of Z Transform.

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